Nested Belief Systems

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AI Seminar Series

Roadmap

- Example scenarios involving nested belief modeling
  - Games with Incomplete Information
  - Multiagent interaction
  - User-modeling in NL systems
- Mathematical representation of nested beliefs
  - Game Theory
  - Multiagent systems - I-POMDPs
- Common Knowledge

Games with Incomplete Information

	Enter	Don't
Build	0,-1	2,0
Don't Build	2,1	3,0

	Enter	Don't
Build	1.5,-1	3.5,0
Don't Build	2,1	3,0

Table 1: Payoffs if 1's cost is **high** and **low** respectively

- When 1's cost is high, *don't build* is the dominant strategy
- When 1's cost is low, 1's optimal strategy depends on the prediction whether 2 enters or not. *Build* turns out to be an optimal action when the prob. that 2 enters < 1/2

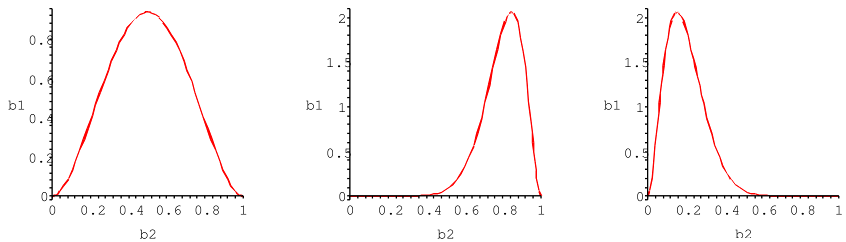
1 must try to predict 2's behavior to choose its own action, which in turn depends on what 2 believes 1's action will be and so on ...

Multiagent Tiger Game

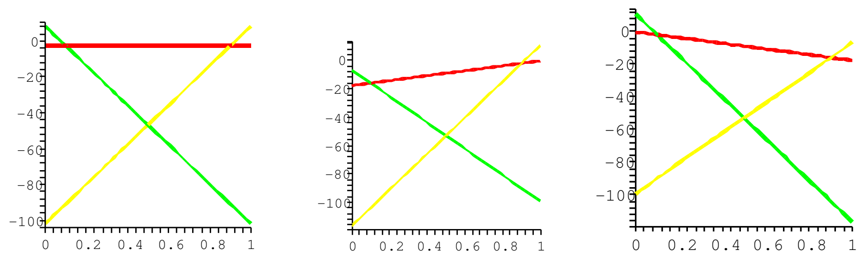
Action/State	TL	TR
$< OR, OR >$	+20	-200
$< OL, OL >$	-200	+20
$< OR, OL >$	-90	-90
$< OL, OR >$	-90	-90

Action/State	TL	TR
$< OR, OR >$	0	0
$< OL, OL >$	0	0
$< OR, OL >$	110	-110
$< OL, OR >$	-110	110

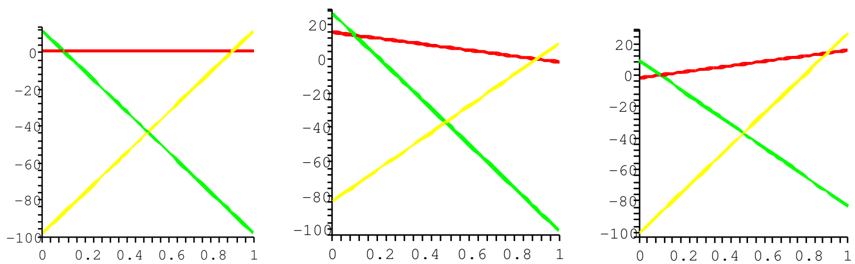
Single-level nesting



Friend



Enemy



User Modeling in NL

- Several NL applications require user modeling
  - NL Dialogue
  - NL Discourse Understanding

- User Modeling $\rightarrow$ Reason about user's mental state
  - mental state $\rightarrow$ beliefs and nestings of beliefs

Mathematical Representation

- Game Theory
  - Summarize the infinite nesting of beliefs in a *type* (Harsanyi)
  - Type Construction:

    $X_0 = S$

    $X_1 = X_0 \times \Delta(X_0) = S \times \Delta(S)$

    $X_2 = X_1 \times \Delta(X_1) = S \times \Delta(S) \times \Delta(S \times \Delta(S))$

    .

    $X_n = X_{n-1} \times \Delta(X_{n-1})$

    type $\theta_i = (b_i^0, b_i^1, b_i^2, ..., b_i^\infty)$ where $b_i^n \in \Delta(X_n)$, and

    $\theta_i \in \times_{n=0}^{\infty} \Delta(X_n)$.
  - Assume **coherency** of types:

    for $n \geq 1 \quad marg_{X_{n-1}} b_i^n = b_i^{n-1}$

    $\sum_{\Delta(X_{n-1})} \Delta(X_{n-1}, \Delta(X_{n-1})) = \Delta(X_{n-1})$

Mathematical Representation

- Multiagent Interaction - I-POMDPs
  - Type Construction:

    $IS_0 = S$

    $IS_1 = S \times \Delta(IS_0) = S \times \Delta(S)$

    $IS_2 = S \times \Delta(IS_1) = S \times \Delta(S \times \Delta(IS_1))$

    .

    $IS_n = S \times \Delta(IS_{n-1})$

    type $\theta_i = (b_i^\infty)$ where $b_i^\infty \in \Delta(IS_\infty)$ with the implicit assumption of coherency.

Common Knowledge

- What does it mean when we say that Event E is common knowledge?
  - An event E is common knowledge (in the population) if all know E, all know that all know it, and so on.
- A formalization of this notion (Aumann)
  - Let N = population, E = event whose occurrence is common knowledge
  - Define K such that $K_i E \Rightarrow$ agent i knows about E, $K_j K_i E \Rightarrow$ agent j knows that agent i knows about E
  - Define $K^1 E = \bigcap_{i \in N} K_i E$, $K^2 E = K^1 K^1 E$, then $K^\infty E = K^1 E \cap K^2 E ...$
  - $K^\infty E$ is the event that E is commonly known

Conclusion

- Nestings (Hierarchies) of beliefs arise in an essential way in several fields such as game theory, and multiagent interaction.
- We must develop rigorous mathematical tools to work with these belief nestings, and discover its properties.
- Empirical results already validate the notion, theoretical results must be uncovered.